EXTRA MATHEMATICS ADMISSIONS TEST

December 2022

Time allowed: 1 hour

Surname	
Other names	

This paper contains 10 multiple choice questions.
Calculators are not permitted.

For each question on pages 2–11 you will be given **five** possible answers, just one of which is correct. Indicate for each question **A-J** which answer (a), (b), (c), (d), or (e) you think is correct with a tick (✓) in the corresponding column in the table below.

	(a)	(b)	(c)	(d)	(e)
A					
B					
C					
D					
E					
F					
G					
H					
I					
J					

A. Whenever I toss a particular coin, it lands on heads with probability $\cos^2 \alpha$ for some fixed real number α (and the outcome is independent of other tosses). I toss the coin three times. The probability that the coin lands on heads two or more times is equal to

(a) $1 + 3\sin^4 \alpha - 2\sin^6 \alpha$,

(b) $1 - 3\sin^4 \alpha - 2\sin^6 \alpha$,

(c) $1 + 3\sin^4 \alpha + 2\sin^6 \alpha$,

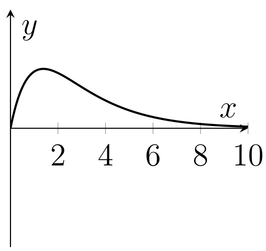
(d) $1 - 3\sin^4 \alpha + 2\sin^6 \alpha$,

(e) $1 + 8\sin^6 \alpha$.

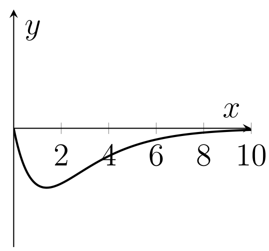
Please turn over

B. Which of the following graphs is a sketch of $e^{-x/2} - e^{-x}$ for $x > 0$?

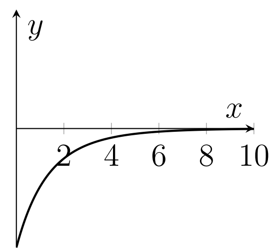
(a)



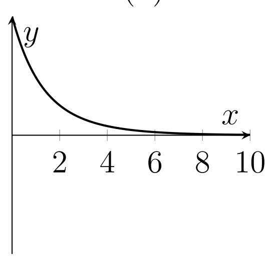
(b)



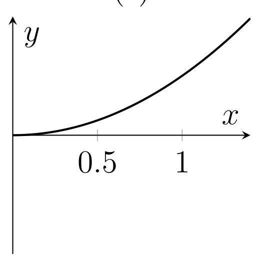
(c)



(d)



(e)



Please turn over

C. For precisely which non-zero real values of x is it true that

$$x^2 - 3x + 2 < \frac{x-1}{x} \quad ?$$

(a) $x < 1 - \sqrt{2}$ or $x > 1 + \sqrt{2}$,

(b) $1 - \sqrt{2} < x < 0$ or $1 < x < 1 + \sqrt{2}$,

(c) $1 < x < 1 + \sqrt{2}$,

(d) $1 - \sqrt{2} < x < 1 + \sqrt{2}$,

(e) $1 - \sqrt{2} < x < 0$.

Please turn over

D. Consider the two inequalities

$$1 \leq x^2 + y^2 \leq 4 \quad \text{and} \quad x^2 \geq 3y^2.$$

The **total** area of all regions of the (x, y)-plane where **both** inequalities hold is

(a) $\pi\sqrt{3}$,

(b) π,

(c) 2π,

(d) $\frac{\pi}{2}$,

(e) $\frac{\pi^2}{6}$.

Please turn over

E. The points $(0, 1)$ and (p, q) are on opposite ends of the diameter of circle C. The x-axis is a tangent to the circle C if and only if

(a) $p = 1 + q$,

(b) $pq = 1$,

(c) $p^2 = 4q$,

(d) $p^2 + (q - 1)^2 = 1$,

(e) $p + q = 1$.

Please turn over

F. The series

$$1 + (1 + x - x^2) + (1 + x - x^2)^2 + (1 + x - x^2)^3 + \dots$$

converges to $\dfrac{1}{x(x-1)}$ for precisely which real values of x?

(a) If and only if $-1 < x < 1$,

(b) If and only if we have both $x \neq 0$ and $x \neq 1$,

(c) If and only if either $-1 < x < 0$ or $1 < x < 2$,

(d) If and only if either $-2 < x < -1$ or $0 < x < 1$,

(e) For all real x.

Please turn over

G. Given that $y = f(x)$ is a solution to $\dfrac{\mathrm{d}y}{\mathrm{d}x} = y^{1/4}$, it follows that one of the following functions is a solution to $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2y^{1/4}$. Which one?

(a) $y = (2)^{-4} f(x)$,

(b) $y = (2)^{3} f(x)$,

(c) $y = (2)^{4/3} f(x)$,

(d) $y = (2)^{-3} f(x)$,

(e) $y = (2)^{4} f(x)$.

Please turn over

H. Suppose that a function $f(n)$ on the positive integers is defined such that $f(1) = 1$ and then for $n \geq 1$

$$f(2n) = f(n) \quad \text{and} \quad f(2n+1) = f(n) + f(n+1).$$

How many values of n are there such that $f(n) = 3$ and also n is a multiple of 35?

(a) 0,

(b) 1,

(c) 2,

(d) 3,

(e) Infinitely many.

Please turn over

I. The number of positive solutions x to the equation

$$\log_2 x = \log_2(x+a) + b,$$

where a and b are non-zero real numbers, is

(a) zero if $ab < 1$, or one if $ab > 1$,

(b) one if $ab < 1$, or two if $ab > 1$,

(c) one if $ab < 0$, or zero if $ab > 0$,

(d) zero if $ab < 0$, or one if $ab > 0$,

(e) one if $ab < 1$, or zero if $ab > 1$.

Please turn over

J. Given that $\int_1^2 \frac{x^2}{1+x^4}\,\mathrm{d}x = A$ where A is some positive real number (which you should not attempt to determine), it follows that the value of $\int_1^2 \frac{x^{-2}}{1+x^4}\,\mathrm{d}x$ is equal to

(a) $1 - A$,

(b) $-A$,

(c) $\frac{1}{A}$,

(d) $A - 1$,

(e) $\frac{1}{2} - A$.

End of last question