EXTRA MATHEMATICS ADMISSIONS TEST

November 2023

Time allowed: 1 hour

Full Name	
UCAS ID	
MAT ID	

This paper contains 10 multiple choice questions.
Calculators are not permitted.

For each question on pages 2–11 you will be given **five** possible answers, just one of which is correct. Indicate for each question **A-J** which answer (a), (b), (c), (d), or (e) you think is correct with a tick (✓) in the corresponding column in the table below.

	(a)	(b)	(c)	(d)	(e)
A					
B					
C					
D					
E					
F					
G					
H					
I					
J					

A. The function $p(x) = x^3$ is an example of a polynomial with the property that there is a point on the graph $y = p(x)$ with zero derivative which is neither a local maximum nor a local minimum. Which one of the following polynomials has the same property?

(a) $y = x^3 - 3x^2 + x$

(b) $y = x^3 - 3x^2 + 2x$

(c) $y = x^3 - 3x^2 + 3x$

(d) $y = x^3 - 3x^2 + 4x$

(e) $y = x^3 - 3x^2 + 5x$

Please turn over

B. Which of the following numbers is the smallest?

(a) $\log_{10}\left(\sqrt[10]{10^{11}}\right)$

(b) $\dfrac{\pi}{2}$

(c) $\dfrac{11}{9}$

(d) $\sqrt{\dfrac{3}{2}}$

(e) $\sqrt{3}\cos(44^\circ)$

Please turn over

C. The sum $\sum_{k=1}^{n} k^3 = \dfrac{n^2(n+1)^2}{4}$. It follows that

$$1^3 + 3^3 + 5^3 + 7^3 + \cdots + 19^3$$

is equal to

(a) 19,800

(b) 19,900

(c) 20,000

(d) 20,100

(e) 20,200

Please turn over

D. All even square numbers are multiples of 4. All odd square numbers are one more than a multiple of 4. It follows that the number of positive integer solutions (x, y) to the equation

$$x^2 + 3y^2 = 4442$$

is

(a) 0

(b) 1

(c) 2

(d) 3

(e) 4442

Please turn over

E. Let $d(n)$ be the number of digits in a positive integer n (with n written in the usual decimal notation). For example $d(2) = 1$, $d(103) = 3$, and $d(10^6) = 7$.

Define the sequence $s_n = (20)^{-d(n)}$. What is the sum $\sum_{n=1}^{\infty} s_n$ equal to?

(a) $\frac{1}{2}$

(b) $\frac{4}{5}$

(c) $\frac{9}{10}$

(d) 1

(e) $\frac{9}{5}$

Please turn over

F. For two vectors $\begin{pmatrix} a \\ b \end{pmatrix}$ and $\begin{pmatrix} c \\ d \end{pmatrix}$ with integer components (positive or negative or zero), we define the function

$$f\left(\begin{pmatrix} a \\ b \end{pmatrix}, \begin{pmatrix} c \\ d \end{pmatrix}\right) = \begin{pmatrix} ac + bd \\ ad + bc + 2bd \end{pmatrix}.$$

How many vectors $\begin{pmatrix} a \\ b \end{pmatrix}$ with integer components are there such that

$$f\left(\begin{pmatrix} a \\ b \end{pmatrix}, \begin{pmatrix} a \\ b \end{pmatrix}\right) = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad ?$$

(a) 0

(b) 1

(c) 2

(d) 3

(e) Infinitely many

Please turn over

G. For a pair of integers x and y with $x \geq 0$ and $y > 0$, we define

$$f(x, y) = \frac{1}{2}(x + y)(x + y + 1) + y$$

What is the set of possible values that $f(x, y)$ can take?

(a) All positive integers.

(b) All positive even integers.

(c) All positive integers except for odd prime numbers.

(d) All positive integers that are triangular numbers (those which are the sum of the first k positive integers for some $k \geq 1$).

(e) All positive integers except for the triangular numbers.

Please turn over

H. Let $p(x) = 2x^4 - 3x^3 - 5x^2 + 2x + 2$. Given that the $y = mx$, with m a real number, crosses the curve $y = p(x)$ at four distinct points, let the x-coordinates of those points be x_1, x_2, x_3, and x_4. The product $x_1x_2x_3x_4$ is equal to

(a) 0

(b) 1

(c) 2

(d) 3

(e) Not enough information

Please turn over

I. Consider the nine lines $y = 2x + 1$, $y = 2x + 2$, $\ldots$, $y = 2x + 9$ and the seven lines $y = -x + 1$, $y = -x + 2$, $\ldots$, $y = -x + 7$.

How many distinct points are there at which the line $y = 1 - 10x$ crosses one or more of the other lines?

(a) 12

(b) 13

(c) 14

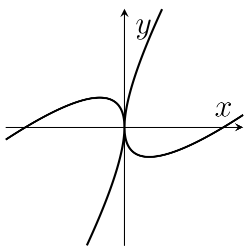
(d) 15

(e) 16

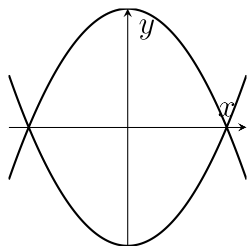
Please turn over

J. Which of the following is the graph of $y(y^3 + 4y^2x + 4x^3) = x^2(1 - x^2 - 6y^2)$?

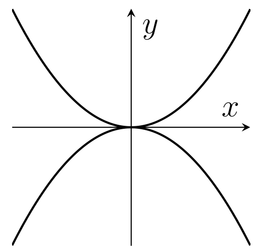
(a)



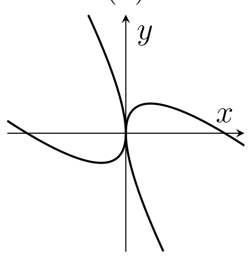
(b)



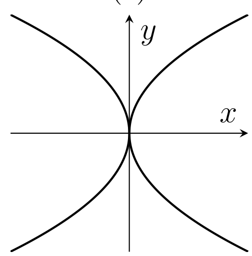
(c)



(d)



(e)



End of last question