PRISM-games

Model Checking for Stochastic Games

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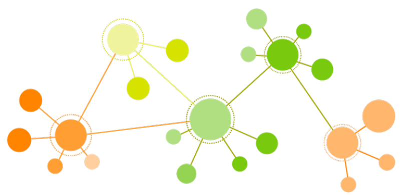
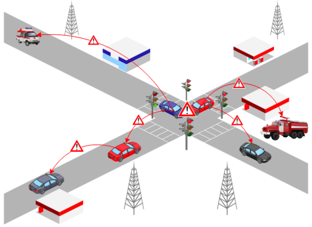
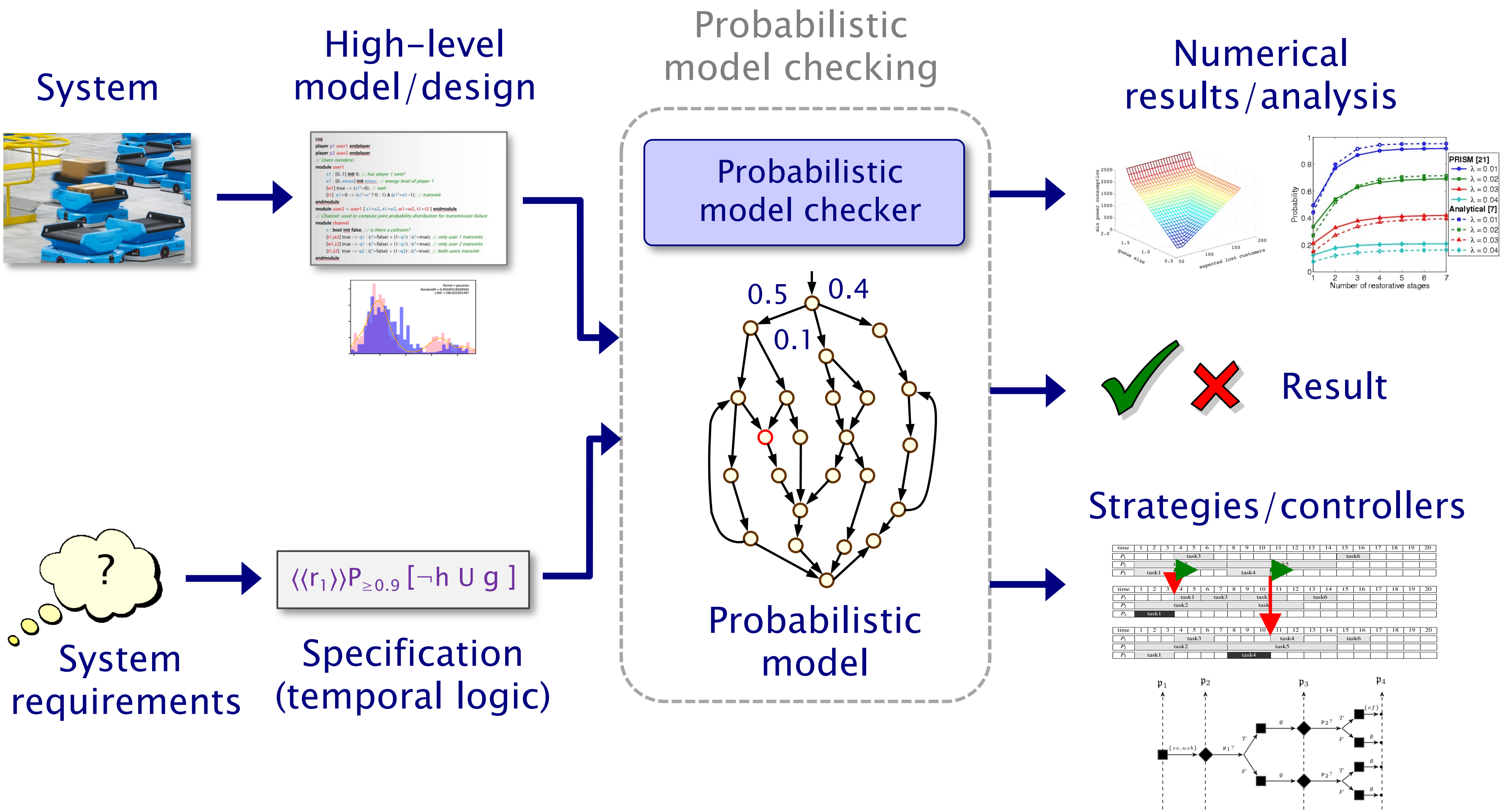
Verification with stochastic games

- How do we formally verify stochastic systems with…
  - multiple autonomous agents acting concurrently
  - competitive or collaborative behaviour between agents, often with differing/opposing goals
  - e.g. security protocols, algorithms for distributed consensus, energy management, autonomous robotics, auctions

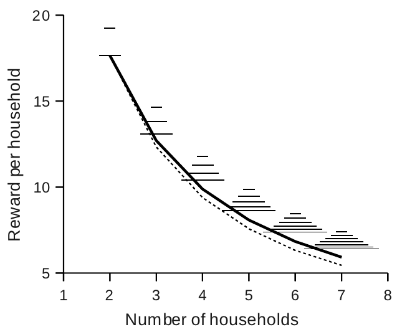

- Probabilistic model checking for stochastic games
  - synthesis and verification of strategies for agents to provide guarantees on safety/performance/… in adversarial settings and stochastic environments

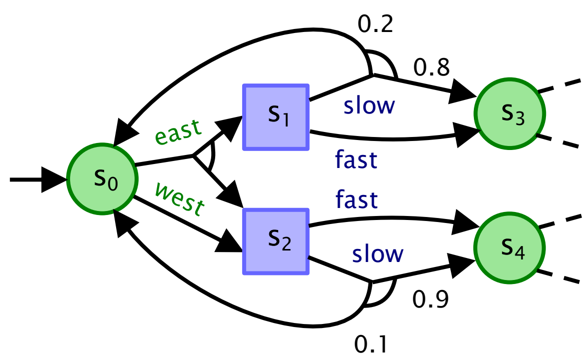
Probabilistic model checking

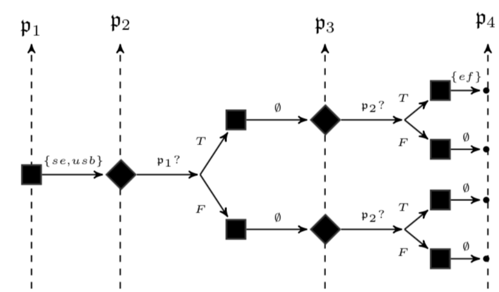


PRISM-games

- PRISM-games: prismmodelchecker.org/games
  - extension of PRISM for stochastic games
  - modelling language + model checking + user interface
  - explicit state & symbolic implementations; simulation







- Example applications (see web site for ~40 case studies)
  - attack-defence trees; network protocols; intrusion detection
  - human-in-the-loop UAV planning; multi-robot systems
  - autonomous driving; self-adaptive software architectures
  - collective decision making; team formation; trust models

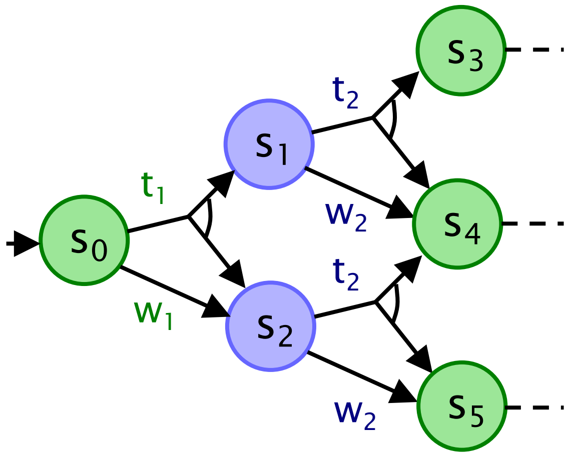
Overview

- Models & modelling
  - stochastic multi-player games
- Property specification
  - temporal logics
- Solving stochastic games
  - algorithms, tools, case studies
  - turn-based/concurrent games
  - zero-sum/equilibria

Models
&
modelling

Stochastic multi-player games

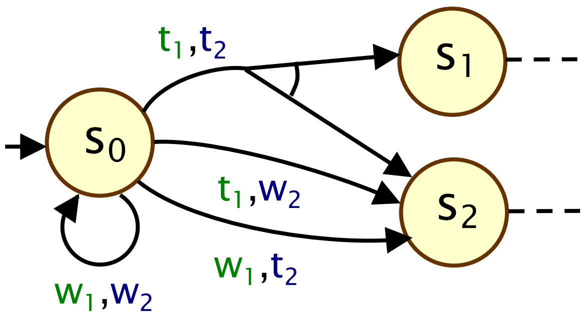
Turn-based stochastic games (TSGs)



- transition function:
  - $\delta : S \times A \to Dist(S)$
- with state partition:
  - $S = S_1 \uplus \ldots \uplus S_n$
- player i controls states S_i

Concurrent stochastic games (CSGs)

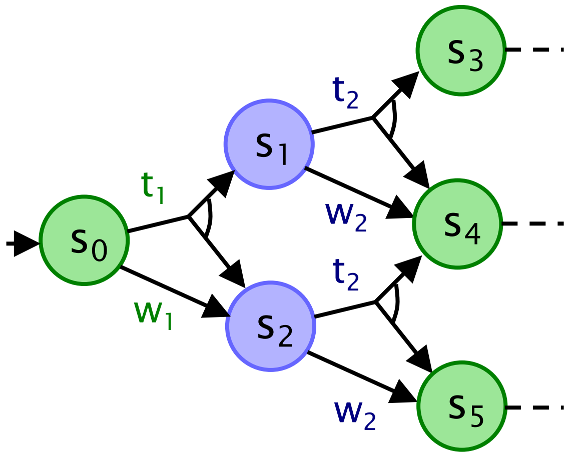
(also: Markov games, multi-agent MDPs)



- transition function:
  - $\delta : S \times (A_1 \cup \{\perp\}) \times \ldots \times (A_n \cup \{\perp\}) \to Dist(S)$
- with joint action space:
  - $A = A_1 \times \ldots \times A_n$
- actions chosen simultaneously

Stochastic multi-player games

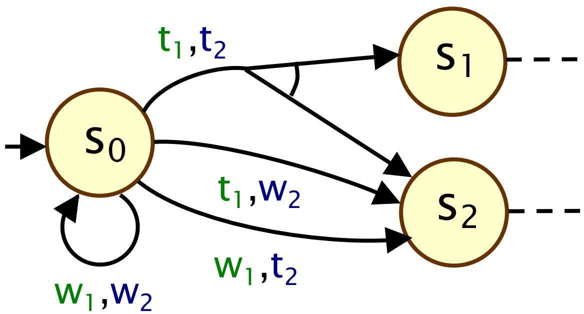
Turn-based stochastic games (TSGs)



- strategies (for player i)
  - $\sigma_i : (S\ A)^*\ S_i \rightarrow \mathrm{Dist}(A)$

Concurrent stochastic games (CSGs)

(also: Markov games, multi-agent MDPs)



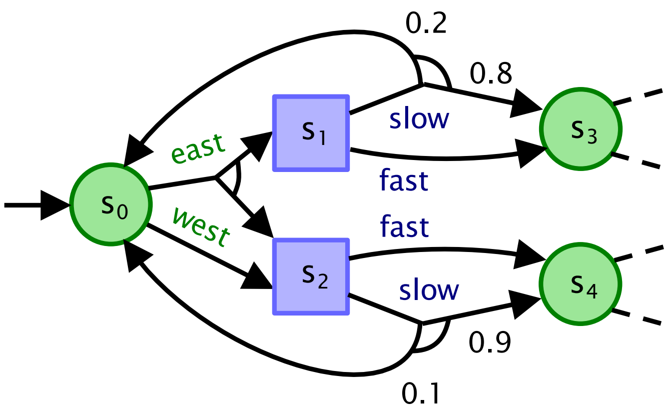
- strategies (for player i)
  - $\sigma_i : (S\ A)^*\ S \rightarrow \mathrm{Dist}(A_i \cup \{\perp\})$

- σ_i can be deterministic/randomised, memoryless/finite-memory/…
- strategy profile $\sigma = (\sigma_1,\ldots,\sigma_n)$ for all n players
- probability space $Pr_s^{\sigma}(\psi)$, or (reward-based) expectation $E_s^{\sigma}(X)$

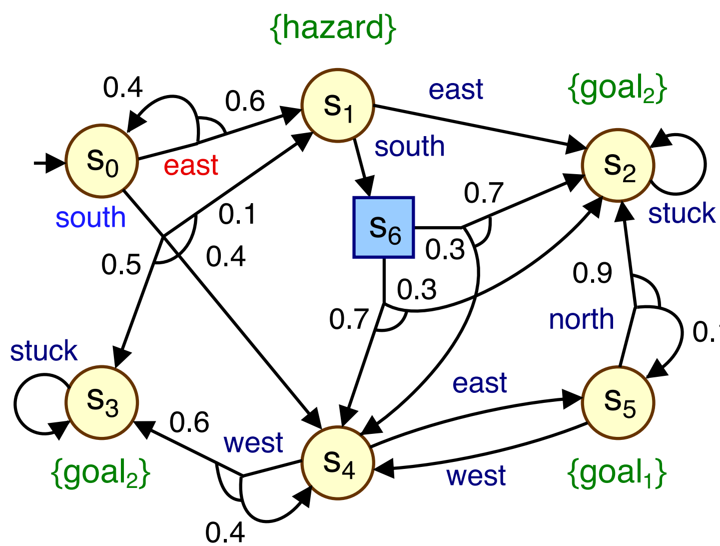
Modelling with turn-based games

- Turn-based stochastic games
  - well suited to some (but not all) scenarios

Shared autonomy:
human-robot control

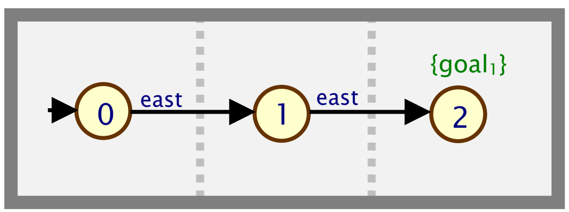


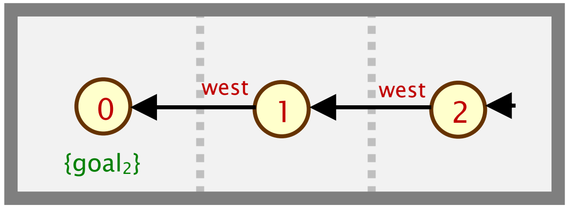
Uncontrollable/unknown
navigation interference

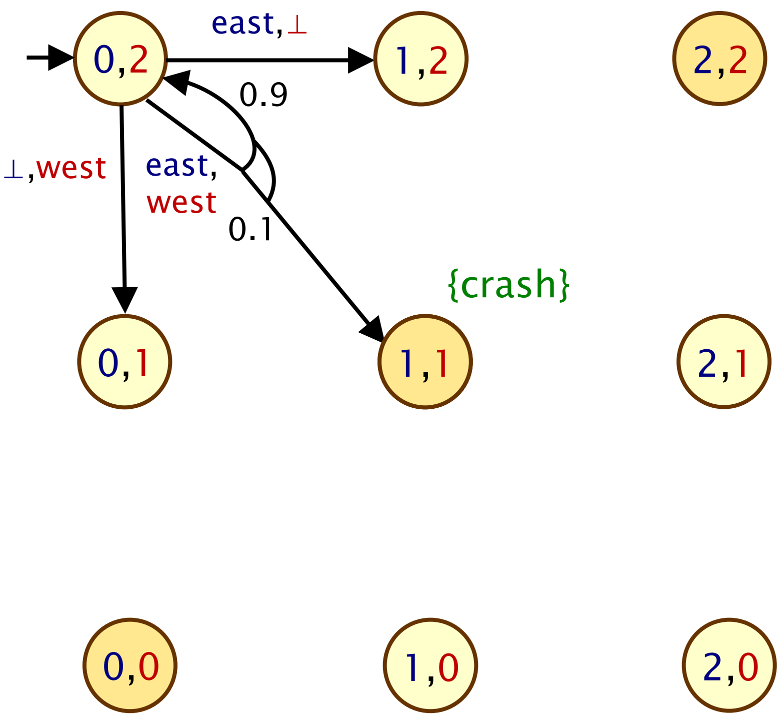


Modelling with concurrent games

- Concurrent stochastic games
  - example: CSG for 2 robots on a 3x1 grid







Modelling with concurrent games

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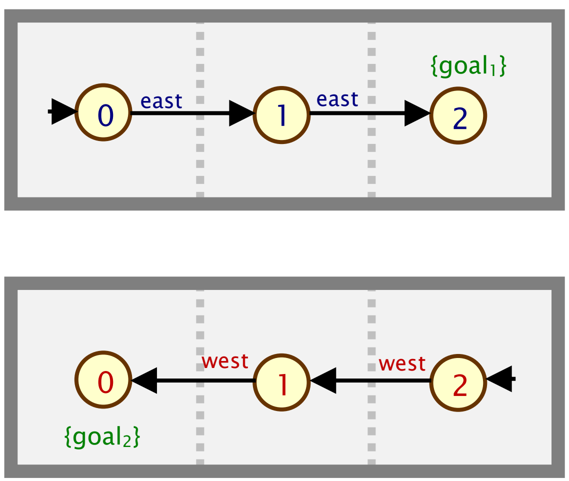

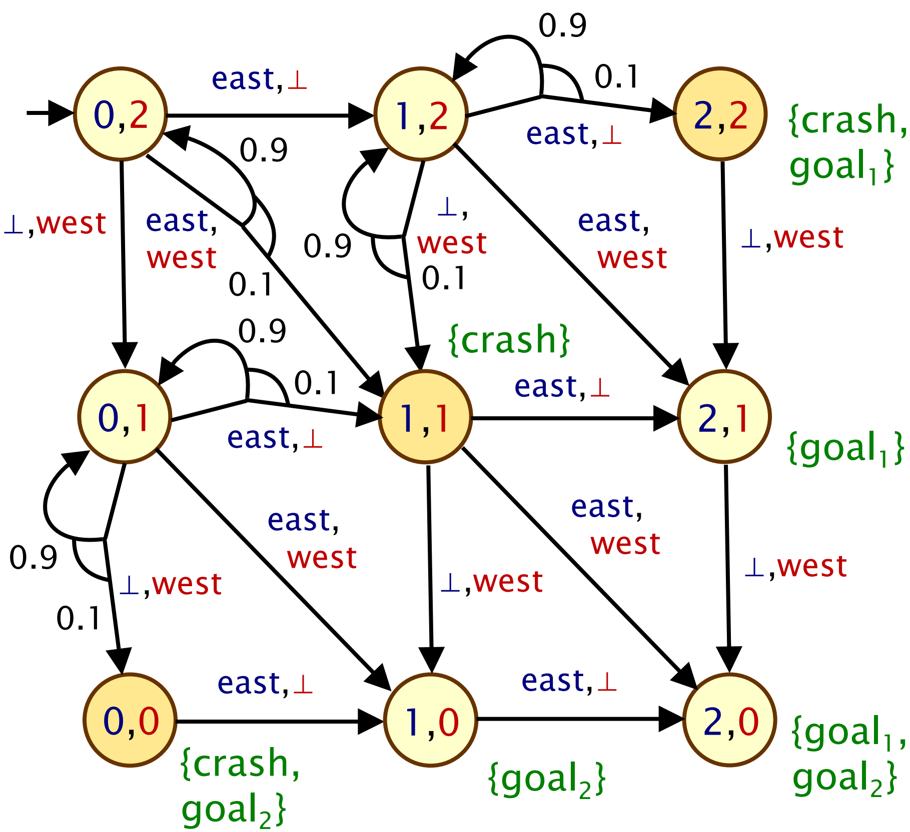

PRISM(-games) modelling language

- PRISM modelling language
  - de-facto standard for probabilistic model checkers
  - key ingredients: modules, variables, guarded commands
  - language features: nondeterminism + probability, parallel composition, costs/rewards, parameters
- PRISM-games modelling language
  - adds: player specifications, joint update distributions

PRISM(-games) modelling language

```
csg
player p1 user1 endplayer
player p2 user2 endplayer
// Users (senders)
module user1
	s1 : [0..1] init 0; // has player 1 sent?
	e1 : [0..emax] init emax; // energy level of player 1
	[w1] true -> (s1'=0); // wait
	[t1]  e1>0 -> (s1'=c' ? 0 : 1) & (e1'=e1-1); // transmit
endmodule
module user2 = user1 [ s1=s2, e1=e2, w1=w2, t1=t2 ] endmodule
// Channel: used to compute joint probability distribution for transmission failure
module channel
	c : bool init false; // is there a collision?
	[t1,w2] true -> q1 : (c'=false) + (1-q1) : (c'=true); // only user 1 transmits
	[w1,t2] true -> q1 : (c'=false) + (1-q1) : (c'=true); // only user 2 transmits
	[t1,t2]  true -> q2 : (c'=false) + (1-q2) : (c'=true); // both users transmit
endmodule
```

Example CSG model (medium access control)

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endmodule
```

A module
is one (parallel)
component

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endmodule
```

Variables define the model state

Guarded commands describe (probabilistic) state updates

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Each player comprises one or more modules

Players have distinct actions, executed simultaneously

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endmodule
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Variable updates can refer to other variables updated simultaneously

Action lists used to specify synchronisation

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  - key ingredients: modules, variables, guarded commands
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- PRISM-games modelling language
  - adds: player specifications, joint update distributions

- Some observations:
  - simple/low-level: no control flow/functions, limited types, …

  \+ uniform language for many types of probabilistic model

  \+ translations exist from more expressive languages

  \+ well suited to symbolic methods (NB: but <u>not</u> to simulation)

Temporal logic

Temporal logic: rPATL

- Temporal logic for stochastic games
  - unambiguous, flexible & tractable behavioural specification
  - basis: rPATL (reward probabilistic alternating temporal logic)
- rPATL is a branching-time logic (extending CTL) with:
  - coalition operator $\langle\langle C \rangle\rangle$ of ATL
  - probabilistic operator P of PCTL
  - generalised (expected) reward operator R from PRISM
  - i.e.: zero-sum, probabilistic reachability + exp. cumul. reward
- Example:
  - $\langle\langle \{r_1,r_3\} \rangle\rangle \, P_{>0.99} [\, F^{\leq 10} (goal_1 \vee goal_3) \,]$
  - "robots 1 and 3 have a strategy to ensure that the probability of reaching a goal location within 10 steps is >0.99, regardless of the strategies of other players"

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  - i.e.: zero-sum, probabilistic reachability + exp. cumul. reward
- Semantics:
  - $s \models \langle\langle C\rangle\rangle P_{\bowtie q}[\psi]$ iff:
  - "there exist strategies for players in coalition C such that, for all strategies of the other players, the probability of path formula ψ being true from state s satisfies ⋈ q"

Temporal logic

- Simple examples (rPATL)

  – Probabilistic reachability
  $\langle\langle r_1 \rangle\rangle\, P_{\geq 0.7}\, [\ F\ \mathit{goal}_1\ ]$
  $\langle\langle r_1 \rangle\rangle\, P_{\geq 0.6}\, [\ F^{\leq 10}\ \mathit{goal}_1\ ]$

  – Probabilistic safety/invariance
  $\langle\langle r_1 \rangle\rangle\, P_{\geq 0.99}\, [\ G \neg \mathit{hazard}\ ]$

  – Probabilistic reach-avoid
  $\langle\langle r_1 \rangle\rangle\, P_{\geq 0.99}\, [\ \neg \mathit{hazard}\ U\ \mathit{goal}_1\ ]$

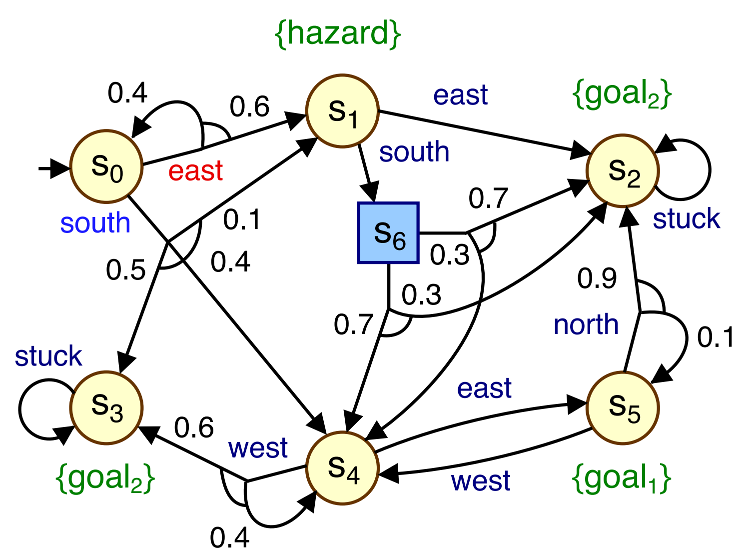
  – Expected cost/reward
  $\langle\langle r_1 \rangle\rangle\, R^{\mathit{steps}}_{\leq 4}\, [\ F\ \mathit{goal}_1\ ]$

  – Numerical ("optimise") queries
  $\langle\langle r_1 \rangle\rangle\, P_{\mathit{max}=?}\, [\ F\ \mathit{goal}_1\ ]$

  – $\langle\langle r_1 \rangle\rangle\, R^{\mathit{time}}_{\mathit{min}=?}\, [\ F\ \mathit{goal}_1\ ]$

Example TSG: robot navigation

(players = robots r_1, r_2)



rPATL and beyond

- Nested specifications in rPATL
  - $\langle\langle\{r_1,r_3\}\rangle\rangle\ R_{min=?}\,[\ \langle\langle\{r_1\}\rangle\rangle\ P_{\geq 0.99}\,[\ F^{\leq 10}\ base\ ]\ U\ (goal_1 \vee goal_3)\ ]$
  - "minimise expected time for joint task between r_1 and r_3, whilst ensuring r_1 can always reliably return to base"

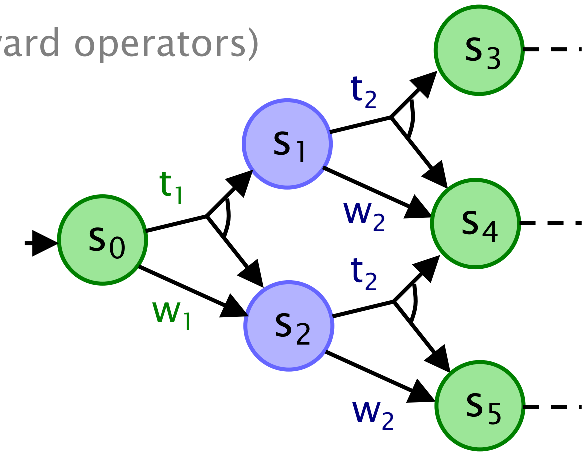
- More expressive temporal specifications
  - e.g. (co-safe) linear temporal logic (LTL)
  - $\langle\langle\{r_1\}\rangle\rangle\ P_{max=?}\,[\ (G\neg hazard) \wedge (GF\ goal_1)\ ]$
  - "maximise the probability visiting $goal_1$ infinitely often and avoiding hazards"

- Non-zero-sum: e.g. Nash equilibria
  - $\langle\langle\{r_1\}{:}\{r_3\}\rangle\rangle\ (R_{min=?}\,[\ F\ goal_1\ ] + R_{min=?}\,[\ F\ goal_3\ ])$
  - "minimise the time to reach the goal for each robot"

Solving stochastic games

Model checking rPATL for TSGs

- Main task: checking individual P and R operators
  - reduces to solving a (zero-sum) 2-player TSG
  - e.g. max/min reachability probability: $\sup_{\sigma_1} \inf_{\sigma_2} Pr_s^{\sigma_1,\sigma_2}(F✓)$
  - optimal strategies are memoryless/deterministic
  - complexity: NP ∩ coNP (if we omit some reward operators)



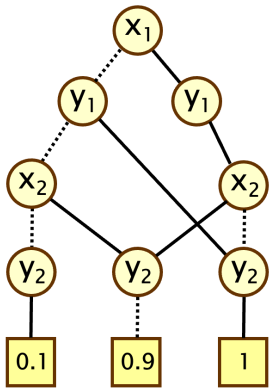
- We use value iteration
  - values p(s) are the least fixed point of:

$$p(s) = \begin{cases} 1 & \text{if } s \models ✓ \\ \max_a \Sigma_{s'} \delta(s,a)(s') \cdot p(s') & \text{if } s \not\models ✓ \text{ and } s \in S_1 \\ \min_a \Sigma_{s'} \delta(s,a)(s') \cdot p(s') & \text{if } s \not\models ✓ \text{ and } s \in S_2 \end{cases}$$

  - and more: graph-algorithms, sequences of fixed points, …

rPATL for TSGs: Implementation

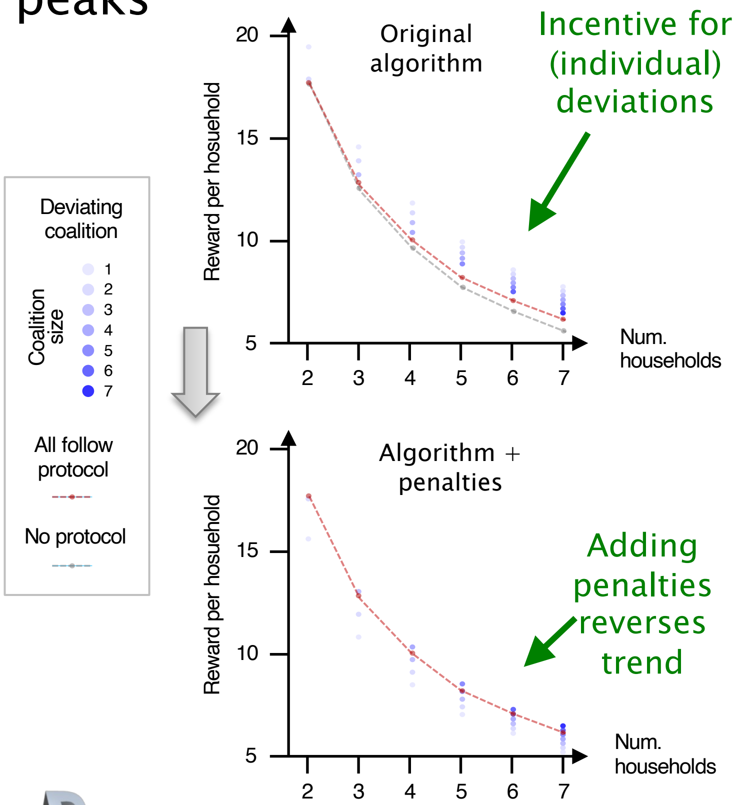
- Value iteration for TSGs
  - similar efficiency and scalability to MDPs
  - (TSGs of, say, 10^7 states easily solvable)
- Also symbolic (BDD-based) implementation
  - exploits model structure/regularity
  - big gains on some models
  - also benefits for strategy compactness

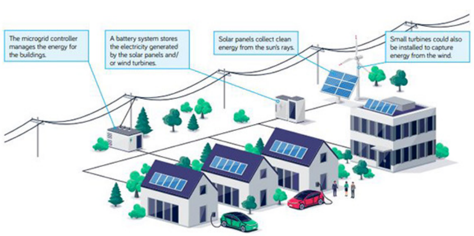


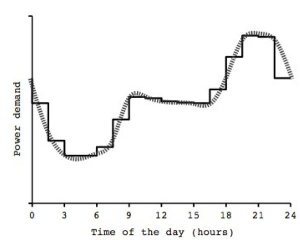
- Other solution methods (and tools) exist
  - strategy iteration, quadratic programming
  - interval/optimistic value iteration (for accuracy guarantees)
  - PRISM-games (and extensions), Tempest, PET, EPMC, …
  - see QComp'23 [ABB+24]

Example: Energy protocols

- Demand management protocol for microgrids [CFK+13b]
  - randomised back-off to minimise peaks
- Stochastic game model + rPATL
  - users can collaboratively cheat (i.e., ignore the protocol)
  - TSGs of up to ~6 million states
  - exposes protocol weakness (incentive to act selfishly)
  - propose/verify simple protocol fix using penalties







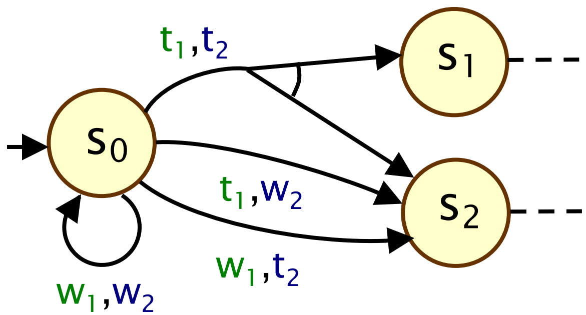
PRISM-games

$\langle\langle C\rangle\rangle R^{rc}{}_{\max=?}\,[\,F^{0}\ \mathrm{end}\,]\ /\ |C|$

Model checking rPATL for CSGs

- Reduces to solving (zero-sum) 2-player CSGs
  - optimal strategies are now randomised (problem is in PSPACE)

- We again use a value iteration based approach
  - e.g. max/min reachability probabilities
  - $\sup_{\sigma_1} \inf_{\sigma_2} \mathrm{Pr}_s^{\sigma_1,\sigma_2}(\mathrm{F}\ \checkmark)$ for all states s
  - values p(s) are the least fixed point of:



$$p(s) = \begin{cases} 1 & \text{if } s \models \checkmark \\ \mathrm{val}(Z) & \text{if } s \not\models \checkmark \end{cases}$$

  - where Z is the matrix game with $z_{ij} = \Sigma_{s'}\ \delta(s,(a_i,b_j))(s') \cdot p(s')$

Model checking rPATL for CSGs

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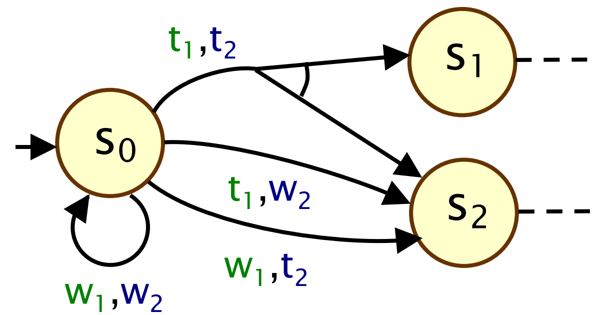
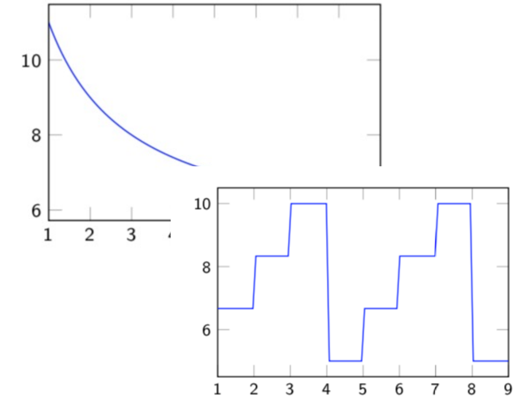


- Implementation
  - need to solve a matrix game at every state and every iteration
  - LP problem of size |A|
  - this is the main performance bottleneck
  - solve CSGs of ~3 million states

Example: Future markets investor

- Model of interactions between:
  - stock market, evolves stochastically
  - two investors i_1, i_2 decide when to invest
  - market decides whether to bar investors

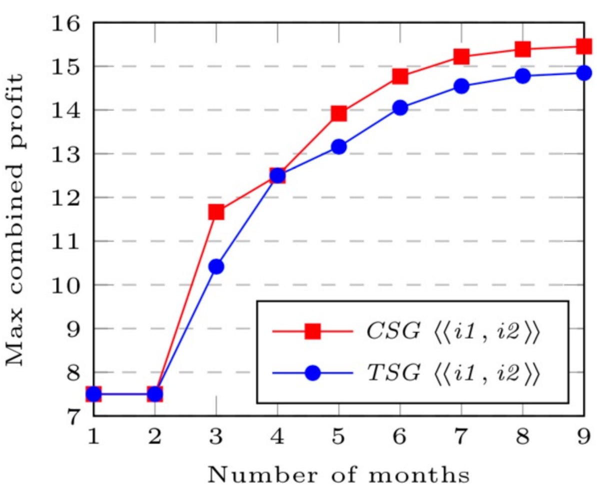


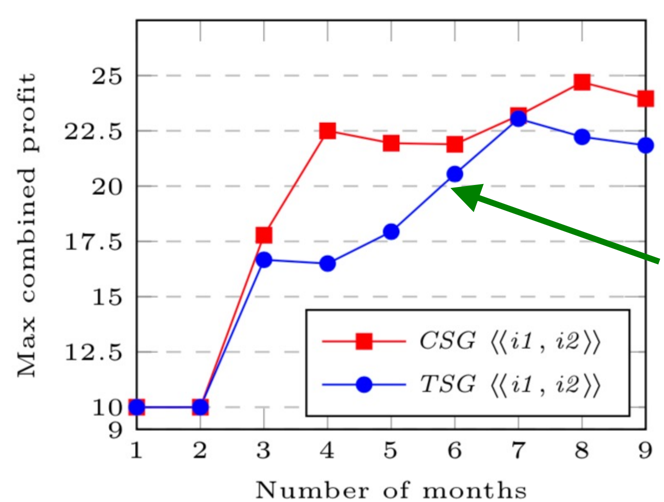
- Modelled as a 3-player CSG
  - extends simpler model originally from [McIver/Morgan'07]
  - investing/barring decisions are simultaneous
  - profit reduced for simultaneous investments
  - market cannot observe investors' decisions

- Analysed with rPATL model checking & strategy synthesis
  - distinct profit models considered: 'normal market', 'later cash-ins' and 'later cash-ins with fluctuation'
  - comparison between TSG and CSG models

Example: Future markets investor

- Example rPATL query:
  - $\langle\langle \mathrm{investor}_1, \mathrm{investor}_2 \rangle\rangle \, \mathrm{R}^{\mathrm{profit}_{1,2}}_{\mathrm{max}=?} \, [\, \mathrm{F}\ \mathrm{finished}_{1,2}\,]$
  - i.e. maximising joint profit
- Results: with (left) and without (right) fluctuations
  - optimal (randomised) investment strategies synthesised
  - CSG yields more realistic results (market has less power due to limited observation of investor strategies)





Too pessimistic: unrealistic strategy for adversary

Equilibria-based properties

Equilibria-based properties

- Non-zero-sum CSGs
  - player objectives are distinct, but not directly opposing
- For now: Nash equilibria (NE) (we will later use other equilibria)
  - no incentive for any player to unilaterally change strategy
  - actually, we use ϵ-NE, which always exist for CSGs
  - a strategy profile $\sigma=(\sigma_{1,\dots,}\sigma_n)$ for a CSG is an ϵ-NE for state s and objectives $X_1,\dots,X_n$ iff:
  - $\mathrm{Pr}_s^{\sigma}(X_i) \geq \sup \{ \mathrm{Pr}_s^{\sigma'}(X_i) \mid \sigma'=\sigma_{-i}[\sigma_i'] \text{ and } \sigma_i' \in \Sigma_i \} - \epsilon$ for all i
  - we use subgame-perfect ϵ-NE, where this holds for all states s
- To formulate the model checking (strategy synthesis) problem, we use social-welfare Nash equilibria (SWNE)
  - these are NE which maximise the sum $E_s^{\sigma}(X_1) + \dots E_s^{\sigma}(X_n)$
  - i.e., optimise the players combined goal

Extending rPATL: Equilibria

- We extend rPATL accordingly:

Zero-sum properties → Equilibria-based properties

$\langle\langle r_1 \rangle\rangle_{max=?}\ P\ [\ F^{\leq k}\ goal_1\ ]$

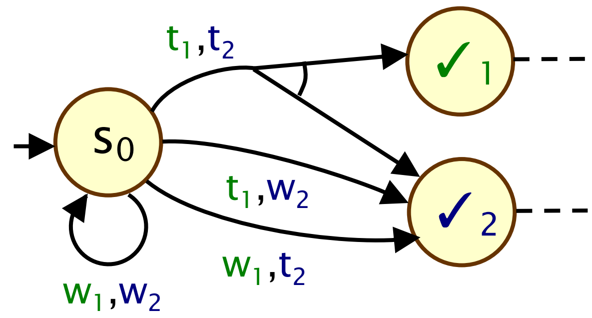
find a robot 1 strategy which maximises the probability of it reaching its goal, regardless of robot 2

$\langle\langle r_1{:}r_2 \rangle\rangle_{max=?}\ (P\ [\ F^{\leq k}\ goal_1\ ]+P\ [F^{\leq k}\ goal_2])$

find (SWNE) strategies for robots 1 and 2 where there is no incentive to change actions and which maximise joint goal probability

Equilibria model checking for CSGs

- Model checking for CSGs with equilibria
  - first: 2-coalition case [KNPS19]
  - we need “stopping game” assumptions
  - requires solution of bimatrix games



- We further extend the value iteration approach:

$$p(s) = \begin{cases} (1,1) & \text{if } s \models \checkmark_1 \wedge \checkmark_2 \\ (p_{max}(s,\checkmark_2),1) & \text{if } s \models \checkmark_1 \wedge \neg\checkmark_2 \\ (1,p_{max}(s,\checkmark_1)) & \text{if } s \models \neg\checkmark_1 \wedge \checkmark_2 \\ val(Z_1,Z_2) & \text{if } s \models \neg\checkmark_1 \wedge \neg\checkmark_2 \end{cases}$$

standard MDP analysis

bimatrix game

  - where Z_1 and Z_2 encode matrix games similar to before

Equilibria model checking for CSGs

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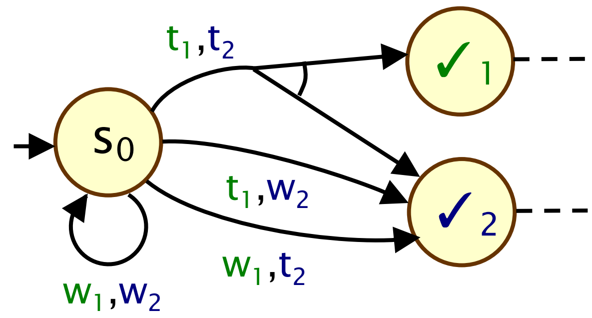
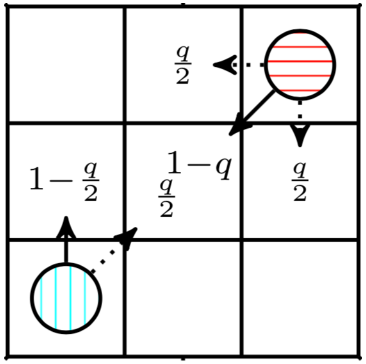


- Implementation
  - we adapt a known approach using labelled polytopes, implemented via SMT
  - optimisations: filtering of dominated strategies
  - solve CSGs of ~2 million states

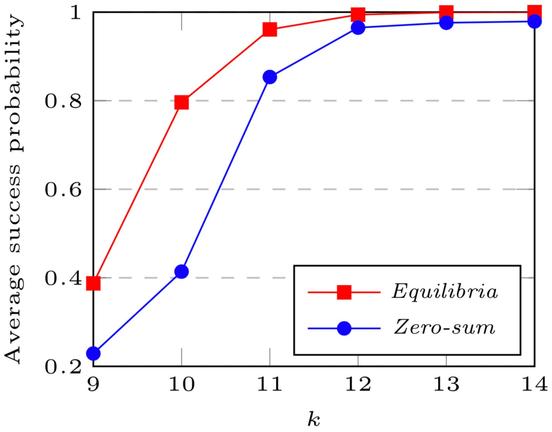
- Extension
  - n-coalition case in [QEST’20]
  - can’t use labelled polytopes
  - needs nonLPs for each state
  - poorer scalability

Example: multi-robot coordination

- 2 robots navigating an N x N grid
  - start at opposite corners, goals are to navigate to opposite corners
  - obstacles modelled stochastically: navigation in chosen direction fails with probability q



- Results (10 x 10 grid)
  - better performance obtained than using zero-sum methods, i.e., optimising for robot 1, then robot 2
  - ϵ-NE found typically have ϵ=0



$\langle\langle robot1{:}robot2 \rangle\rangle_{max=?} (P [F^{\leq k} goal_1] + P [F^{\leq k} goal_2])$

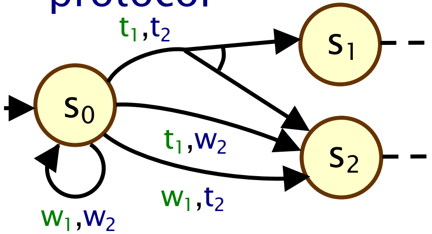
Faster and fairer equilibria

- Limitations of (social welfare) Nash equilibria for CSGs:

1. can be computationally expensive, especially for >2 players
2. social welfare optimality is <u>not</u> always equally beneficial

- Correlated equilibria
  - shared (probabilistic) signal + map to local strategies
  - synthesis: support enumeration + LP (>2 players needs nonLP for NE)
  - much faster to synthesise (4–20x faster)

Example: Aloha communication protocol



Signals: randomised coordination of next message sender, adapting over time

- Social fairness
  - alternative optimality criterion: minimise difference in objectives
  - applies to both Nash/correlated: slight changes to optimisation

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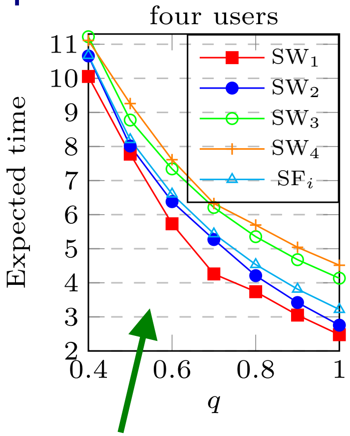


social fairness (SF) more equitable than social welfare (WF$_i$)

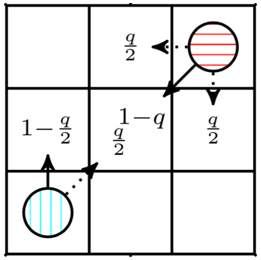
Wrapping up

Summary

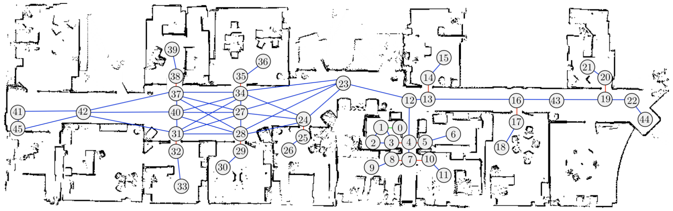
- Probabilistic model checking for stochastic games
  - turn-based and concurrent stochastic games
  - tools for modelling, construction & analysis of large games
  - temporal logics for property specification
  - value iteration based verification and strategy synthesis
  - wide range of interesting application domains & queries

Challenges & directions

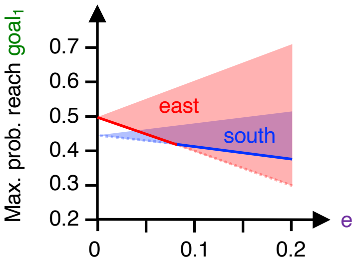
- Partial information/observability
  - needed for practical applications
  - POSGs? DEC-POMDPs?



- Other game theory tools
  - e.g. Stackelberg equilibria



- Managing model uncertainty
  - learning + robust verification



- Accuracy of model checking results
  - value iteration improvements; exact methods
- Scalability & efficiency
  - e.g. symbolic methods, abstraction, symmetry reduction
  - sampling-based strategy synthesis methods

PRISM-games

- See the PRISM-games website for more info
  - prismmodelchecker.org/games/
  - documentation, examples, case studies, papers
  - downloads:
  - open source (GPLV2):