Tractable Extensions of the Description Logic $\mathcal{EL}$ with Numerical Datatypes

Despoina Magka, Yevgeny Kazakov and Ian Horrocks

Oxford University Computing Laboratory

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Outline

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- Polynomial-time reasoning

$\mathcal{EL}^{\perp}$ with Numerical Datatypes

- Concept constructors

	Syntax	Semantics
Concept name	C	$C(x)$
Top	$\top$	$\top$
Bottom	$\perp$	$\perp$
Conjunction	C ⊓ D	$C(x) \wedge D(x)$
Existential restriction	∃R.C	$\exists y : R(x,y) \wedge C(y)$
Datatype restriction	∃F.*range*	$\exists v : F(x,v) \wedge v \in range$

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- Reasoning problems

Classification: compute all A $\sqsubseteq$ B such that $\mathcal{O} \models$ A $\sqsubseteq$ B

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- EL Profile of OWL 2 admits only equality

- Absence of inequalities reduces the utility of OWL 2 EL

Motivating example

Example

$$\text{Panadol} \sqsubseteq \exists \text{contains}.(\text{Paracetamol} \sqcap \exists \text{mgPerTablet}.[= 500])$$

$$\text{Patient} \sqcap \exists \text{hasAge}.[< 6] \sqcap \exists \text{hasPrescription}.$$
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$\text{X} \sqsubseteq \text{Patient} \sqcap \exists \text{hasAge}.[= 3] \sqcap \exists \text{hasPrescription}.\text{Panadol}$

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- Inequalities are used to trigger a rule
- The EL Profile of OWL 2 does not allow inequality relations

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- Main results
  - Full classification of tractable cases for $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$ and $\mathbb{R}$
  - Polynomial, sound and complete reasoning procedure for extensions of $\mathcal{EL}^{\perp}$ with “safe” numerical datatypes

Outline

Algorithm Stages

1. Standard normalization of the axioms

Normal forms	
NF1	$A \sqsubseteq B$
NF2	$A_1 \sqcap A_2 \sqsubseteq B$
NF3	$A \sqsubseteq \exists R.B$
NF4	$\exists R.B \sqsubseteq A$
NF5	$A \sqsubseteq \exists F.\mathit{range}$
NF6	$\exists F.\mathit{range} \sqsubseteq A$

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2. Saturation of the axioms under inference rules, such as:

$$\frac{A \sqsubseteq B \quad A \sqsubseteq C}{A \sqsubseteq D} \quad B \sqcap C \sqsubseteq D \in \mathcal{O}$$

Reasoning Rules (part I)

Rules from $\mathcal{EL}^{\bot}$

IR1 $\dfrac{}{A \sqsubseteq A}$ **IR2** $\dfrac{}{A \sqsubseteq \top}$ **CR1** $\dfrac{A \sqsubseteq B}{A \sqsubseteq C}$ $B \sqsubseteq C \in \mathcal{O}$

CR2 $\dfrac{A \sqsubseteq B \quad A \sqsubseteq C}{A \sqsubseteq D}$ $B \sqcap C \sqsubseteq D \in \mathcal{O}$

CR3 $\dfrac{A \sqsubseteq B}{A \sqsubseteq \exists R.C}$ $B \sqsubseteq \exists R.C \in \mathcal{O}$

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CR5 $\dfrac{A \sqsubseteq \exists R.B \quad B \sqsubseteq \bot}{A \sqsubseteq \bot}$

Reasoning Rules (part II)

New rules for datatypes

$$\frac{\mathsf{A} \sqsubseteq \exists \mathsf{F}.[< m]}{\mathsf{A} \sqsubseteq \mathsf{B}} \quad \exists \mathsf{F}.[< n] \sqsubseteq \mathsf{B} \in \mathcal{O},\ m \leq n$$

Reasoning Rules (part II)

New rules for datatypes

ID1 $\dfrac{}{\mathsf{A} \sqsubseteq \bot}$ $\mathsf{A} \sqsubseteq \exists \mathsf{F}.[< 0] \in \mathcal{O}$

CD1 $\dfrac{\mathsf{A} \sqsubseteq \mathsf{B}}{\mathsf{A} \sqsubseteq \exists \mathsf{F}.\mathit{range}}$ $\mathsf{B} \sqsubseteq \exists \mathsf{F}.\mathit{range} \in \mathcal{O}$

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  - not complete in general
  - complete under certain restrictions on datatypes

Restrictions for $\mathbb{N}$

Negative relations	Positive relations
$<, \leq, >, \geq, =$	$=$

Example

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$<, \leq, >, \geq, =$	$=$
$=$	$<, \leq, >, \geq, =$
$<, \leq$	$<, \leq, >, \geq, =$
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$$\exists \mathsf{F}.[= 0] \sqsubseteq \mathsf{C}$$
$$\overline{\mathsf{A} \not\sqsubseteq \mathsf{B} \sqcup \mathsf{C}}$$

Restrictions for $\mathbb{Q}$ and $\mathbb{R}$

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$<, \leq, >, \geq, =$	$=$
$\leq, =$	$<, \leq, >, \geq, =$
$\geq, =$	$<, \leq, >, \geq, =$
$<, \leq$	$<, \leq, >, \geq, =$
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$<, \leq, =$	$<, >, \geq, =$
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- Between every two different numbers there exists a third one:

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$$\exists \mathsf{F}.[< 2] \sqsubseteq \mathsf{C}$$
$$\overline{}$$
$$\mathsf{A} \not\sqsubseteq \mathsf{B} \sqcup \mathsf{C}$$

Outline

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