Acyclicity Conditions and their Application to Query Answering in Description Logics

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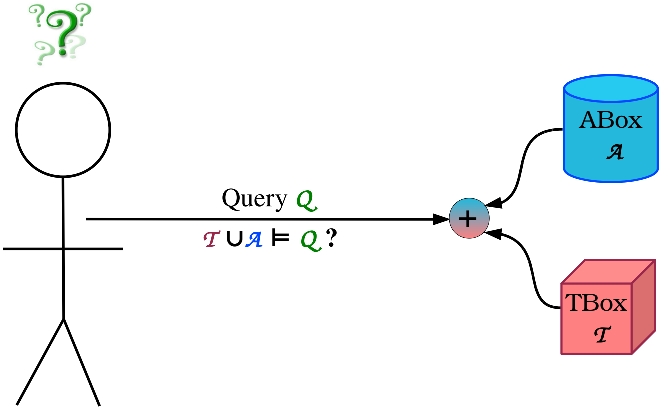
OUTLINE

Ontological Query Answering

- Key reasoning task for DL and rule-based applications



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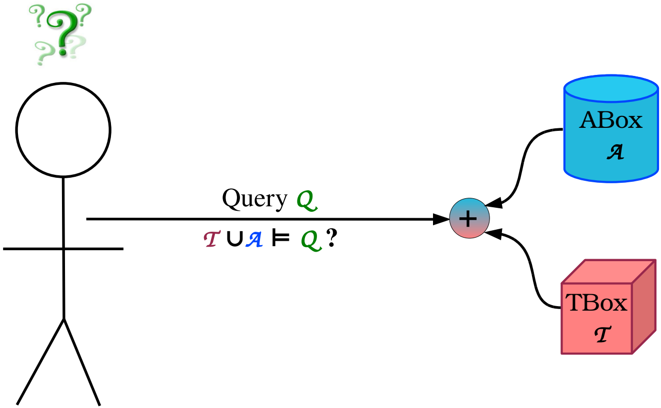

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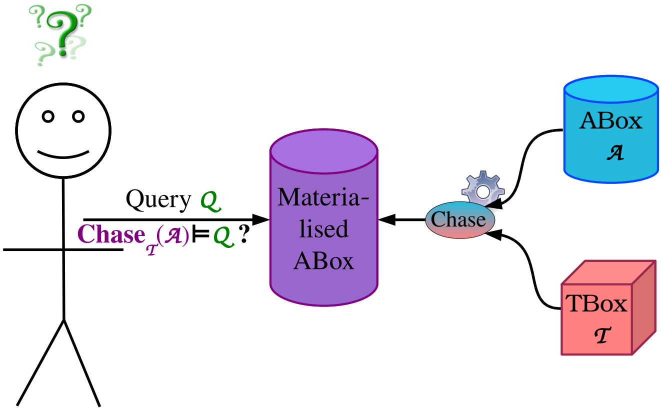
- Answering CQs over DLs $\rightsquigarrow$ high computational complexity
- Materialisation-based paradigm: chase ABox using TBox and evaluate Q in the computed ABox



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2. Data transformation rules in data exchange
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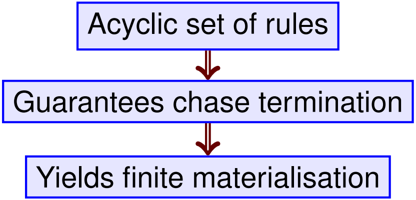

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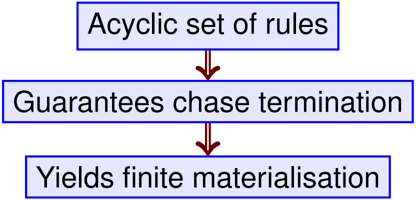

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II. Acyclicity conditions might be too restrictive

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- Suggestion: materialise ABoxes only over acyclic TBoxes
  - Always complete ✓
  - Provably terminating ✓

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Materialisation-based reasoning beyond OWL 2 RL might be practically feasible

Outline

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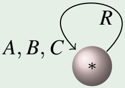

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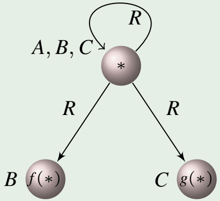

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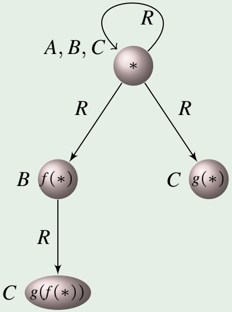

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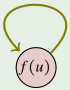

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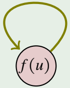

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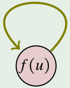

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$$\text{Pos}_B(u) = \{A|_1\} \subseteq \text{Move}(f(u)) = \{R|_2,\ B|_1,\ R|_1,\ A|_1\}$$

- Joint acyclicity
  1. Tracks value generation and propagation to detect cyclic creation of terms
  2. Polynomial time to check
- May overestimate rule applicability

Model-faithful Acyclicity

- Track rule applications more 'faithfully'

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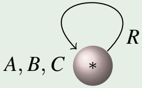

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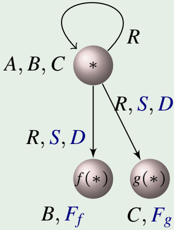

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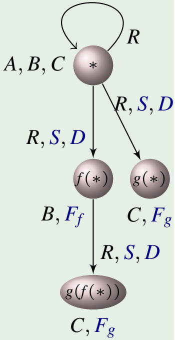

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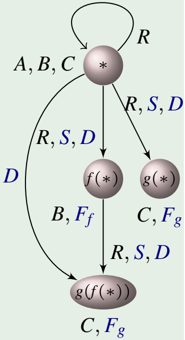

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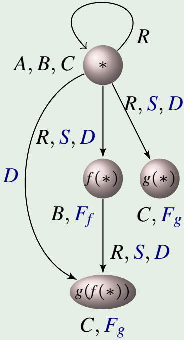

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Cost of Checking MFA

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- Isn't computational complexity too high?

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- Track rule applications just 'faithfully' enough

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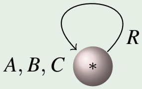

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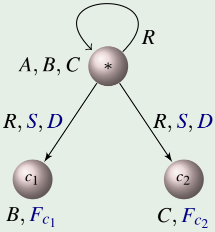

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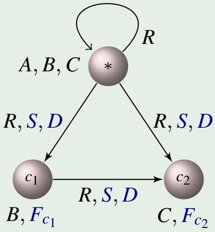

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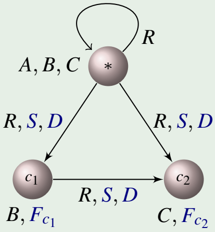

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- Horn-$\mathcal{SHIQ}$ TBoxes can be checked in PTIME for MSA before *potential* materialisation-based query answering

ACYCLICITY CONDITIONS (PARTIAL) TAXONOMY

JA $\subsetneq$ SWA MSA MFA

Acyclicity Conditions (Partial) Taxonomy

- Our contributions:

JA	$\subsetneq$	SWA		MSA		MFA

ACYCLICITY CONDITIONS (PARTIAL) TAXONOMY

- Our contributions:
  1. MSA strictly subsumes SWA

$$\text{JA} \subsetneq \text{SWA} \subsetneq \text{MSA} \qquad \text{MFA}$$

Acyclicity Conditions (Partial) Taxonomy

- Our contributions:
  1. MSA strictly subsumes SWA
  2. MFA strictly subsumes MSA

$$\text{JA} \subsetneq \text{SWA} \subsetneq \text{MSA} \subsetneq \text{MFA}$$

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$$A(x) \rightarrow \exists y.R(x,y) \wedge B(y)$$
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MFA but not MSA

Acyclicity Conditions (Partial) Taxonomy

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  1. MSA strictly subsumes SWA
  2. MFA strictly subsumes MSA

JA $\subsetneq$ SWA $\subsetneq$ MSA $\subsetneq$ MFA

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MFA but not MSA

- MSA and MFA coincide in experimental evaluation of DL ontologies

Outline

Translating DLs into rules

- Axioms of normalised Horn-$\mathcal{SRIQ}$ ontologies can be converted to (existential) rules

$A \sqsubseteq \exists R.B$	$A(x) \to \exists y.R(x,y) \land B(y)$
$A \sqsubseteq \leq 1\, R.B$	$A(z) \land R(z,x_1) \land B(x_1) \land R(z,x_2)$
	$\land B(x_2) \to x_1 \approx x_2$
$A \sqcap B \sqsubseteq C$	$A(x) \land B(x) \to C(x)$
$A \sqsubseteq \forall R.B$	$A(z) \land R(z,x) \to B(x)$
$R \sqsubseteq S$	$R(x_1,x_2) \to S(x_1,x_2)$
$R \circ S \sqsubseteq T$	$R(x_1,z) \land S(z,x_2) \to T(x_1,x_2)$

TRANSLATING DLS INTO RULES

- Axioms of normalised Horn-$\mathcal{SRIQ}$ ontologies can be converted to (existential) rules

$A \sqsubseteq \exists R.B$	$A(x) \rightarrow \exists y.R(x,y) \wedge B(y)$
$A \sqsubseteq \leq 1\, R.B$	$A(z) \wedge R(z,x_1) \wedge B(x_1) \wedge R(z,x_2)$
	$\wedge\ B(x_2) \rightarrow x_1 \approx x_2$
$A \sqcap B \sqsubseteq C$	$A(x) \wedge B(x) \rightarrow C(x)$
$A \sqsubseteq \forall R.B$	$A(z) \wedge R(z,x) \rightarrow B(x)$
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- Equality is handled with a modification of the singularisation [Marnette, PODS, 2009] technique

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   $\mathcal{T}$ is MFA
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2. Horn-$\mathcal{SRI}$ TBox $\mathcal{T}$ and ABox $\mathcal{A}$
   $\mathcal{T}$ is weakly acyclic
   F set of facts

   $\rightsquigarrow$ Deciding $\mathcal{T} \cup \mathcal{A} \models F$ is EXPTIME-hard

Outline

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< 100	70	64	64	64
100–1K	33	30	30	23
1K–5K	20	14	14	12
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12K–160K	12	5	3	3
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- 7 large and expressive OBO ontologies MSA but not JA (only two of them were $\mathcal{ELH}^r$ and DL-Lite)

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Depth	#	generated size		materialisation size	
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Depth $=$ length of function symbol nesting

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SUMMARY OF THE RESULTS

1. More general acyclicity conditions: MSA and MFA
2. Complexity analysis for checking MSA and MFA

	Horn-$\mathcal{SHIQ}$	bounded arity	no restriction
MSA	PTime-complete	coNP-complete	ExpTime-complete
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3. DL query answering under acyclicity conditions
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   - Horn-$\mathcal{SHIQ}$ $\mathcal{T}$ in MFA: $\mathcal{T} \cup \mathcal{A} \models Q$ is PSpace-complete
4. Experimental evaluation on DL ontologies
   - 83% ontologies found acyclic (78% JA)
   - materialised ABoxes not too large $\rightsquigarrow$ $\times$ 5 bigger on average for ontologies with depth < 5 (= most ontologies)

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Thank you! Questions?!?