Substitutes Markets with Budget Constraints: Solving for Competitive *and* Optimal Prices

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Abstract. Markets with multiple divisible goods have been studied widely from the perspective of revenue and welfare. In general, it is well known that envy-free, revenue-maximal outcomes can result in lower welfare than competitive equilibrium. In our market, buyers have quasilinear utilities with linear substitutes valuations and budget constraints, and the seller must find prices and an envy-free allocation that maximise revenue or welfare. This mirrors markets such as ad auctions and auctions for the exchange of financial assets.

We prove that the unique competitive equilibrium prices are also envy-free revenue-maximal. This coincidence of maximal revenue and welfare is surprising and breaks down even when buyers have piecewise-linear valuations. We present a novel characterisation of the set of 'feasible' prices (no excess demand), a non-convex set that we show to exhibit the lower semi-lattice structure. We demonstrate that elementwise-minimal prices maximise revenue and welfare. To prove welfare optimality, we adapt an existing algorithm for Fisher markets. Our procedure scales down any non-minimal feasible prices, maintaining feasibility, thus providing an algorithm for finding this unique price vector.

Our market is also called a 'quasi-Fisher' market. In contrast to standard Fisher markets, buyers spend nothing if prices are too high, making revenue maximisation an interesting objective for the seller. The market is also equivalent to an 'arctic product-mix auction' with zero seller costs, an auction developed to exchange financial assets with limited supply in Iceland.

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