Models of Computation

Tutorial Exercises 1

1. The alphabet is $\{0, 1\}$. Give DFAs that recognise the following languages:
   (i) The empty set.
   (ii) The singleton set consisting of the empty string.
   (iii) All strings except the empty string.
   (iv) $\{ w : w$ starts with 0 and has odd length, or starts with 1 and has even length $\}$
   (v) $\{ w : w$ contains at least two 0s and at most one 1 $\}$
   (vi) $\{ w : w$ contains an even number of 0s, or exactly two 1s $\}$

2. Let L_1 and L_2 be regular languages over the alphabet Σ.
   (i) Using the product construction on DFAs, give a detailed proof that $L_1 \cap L_2$ is regular.
   (ii) Prove that $L_1 \setminus L_2 \stackrel{\text{def}}{=} \{ w \in L_1 : w \notin L_2 \}$ is regular.

3. The alphabet is $\{0, 1\}$. Give NFAs with the specified number of states recognising the following languages:
   (i) The language $\{ w : w$ ends with 00 $\}$ with three states.
   (ii) The language $\{ \epsilon \}$ with one state.
   (iii) The language $\{ 0 \}$ with two states.
   (iv) All words that start and end with the same symbol, with four states.

4. The alphabet is $\{0, 1, \cdots, 9\}$. Give NFAs that recognise the following languages:
   (i) The set of strings such that the final digit has appeared before.
   (ii) The set of strings such that the final digit has *not* appeared before.

5. Design NFAs for the following languages.
   (i) The set of strings consisting of zero or more a's, followed by zero or more b's, followed by zero or more c's.
   (ii) The set of strings of 0's and 1's such that at least one of the last ten positions is a 1.

6. (i) Let M be an NFA that recognises language L. Does swapping the accepting and non-accepting states in M necessarily yield an NFA? Does the new NFA so obtained necessarily recognise the complement of L?
   (ii) Are regular languages closed under complement? Justify your answer.

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