Quantum Processes and Computation

Assignment 1, Monday 21 Oct 2024

Deadline: Class in week 3 (Ask your teacher for weekly marking deadline.)

Goals: After completing these exercises successfully you should be able to perform simple diagrammatic and concrete computations in the process theories of **functions** and **linear maps**.

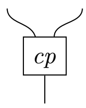
Note: Some of these exercises also appear in *Picturing Quantum Processes*, but sometimes they have been slightly modified for the problem sheet and/or to fit the notations used in the lectures. The corresponding exercise number from the book is shown in brackets.

Exercise 1 (3.12): Give the diagrammatic equations of a process $*$ taking two inputs and one output that express the algebraic properties of being

1. associative: $x * (y * z) = (x * y) * z$
2. commutative: $x * y = y * x$
3. having a unit: there exists a process e (with no inputs) such that $x * e = e * x = x$

Note: x, y and z should not appear in your final diagrams. They are however useful in trying to figure out what the diagrammatic equation should be.

Exercise 2 (3.15): Using the copy operation:

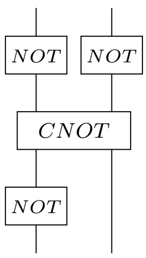


$$:: \quad x \mapsto (x, x)$$

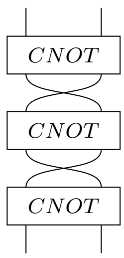
write down the diagram representing distributivity: $(x + y) * z = (x * z) + (y * z))$. Here, $+$ and $*$ are processes that take two inputs and and one output.

Exercise 3 (3.30): First compute the values of the following functions, then show that they can both be expressed by simpler diagrams:

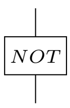
(a) :



(b) :

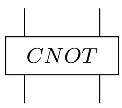


where:



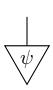
$$:: \begin{cases} 0 \mapsto 1 \\ 1 \mapsto 0 \end{cases}$$

and



$$:: \begin{cases} (0,0) \mapsto (0,0) \\ (0,1) \mapsto (0,1) \\ (1,0) \mapsto (1,1) \\ (1,1) \mapsto (1,0) \end{cases}$$

Exercise 4 (5.54): Let

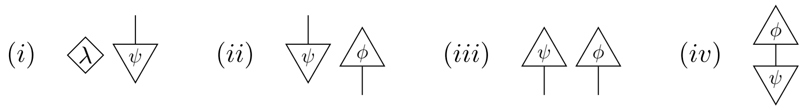


$$= \begin{pmatrix} \psi^0 \\ \psi^1 \end{pmatrix}$$

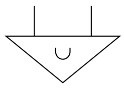
and

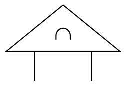
$$= \begin{pmatrix} \phi_0 & \phi_1 \end{pmatrix}$$

be respectively a 2-dimensional state, and 2-dimensional effect in the process theory of **linear maps**. Let λ be a number. Compute the matrices for the following processes

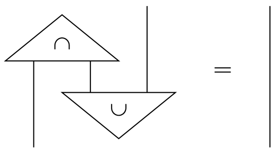


Exercise 5 (5.58): The matrices for cups and caps in 2 dimensions are:


$$= \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

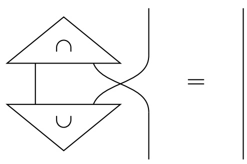

$$= \begin{pmatrix} 1 & 0 & 0 & 1 \end{pmatrix}$$

Verify the yanking equation

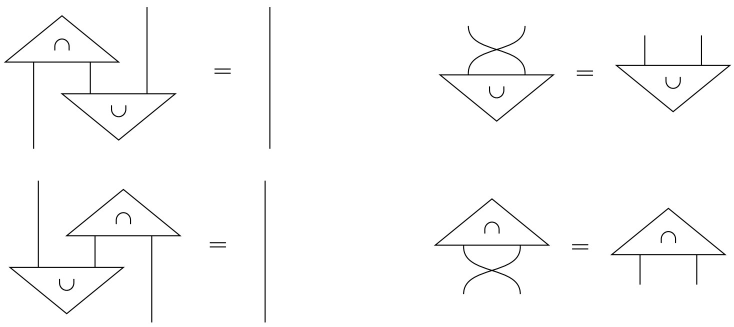


directly using the matrices of the 2-dimensional cup and cup by using the rules for sequential and parallel composition of matrices, i.e. show that $(\cap \otimes 1_{\mathbb{C}^2}) \circ (1_{\mathbb{C}^2} \otimes \cup) = 1_{\mathbb{C}^2}$ (where $1_{\mathbb{C}^2}$ is the 2×2 identity matrix).

Exercise 6 (4.12): Prove that



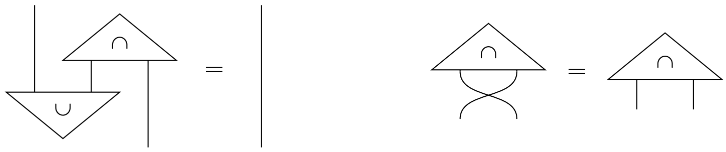
follows from the following 4 equations:



Exercise 7 (4.14 in online version of PQP): Show that, in fact, we only need two equations for caps and cups. Namely, the following are equivalent:

(i) a state and an effect satisfying:

(ii) a state and an effect satisfying:



So in particular, if either eqs. (i) or eqs. (ii) hold, then all equations hold.

Hint: Most people find this (deceptively) difficult. If you are stuck proving (i) $\implies$ (ii), start by thinking about what you can do with this picture: