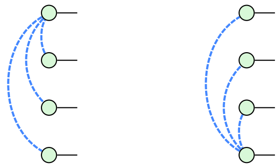
Quantum Software

Assignment 6, Michaelmas 2024

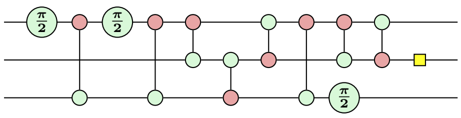
Exercise 1: We say two n-qubit quantum states $|\psi_1\rangle$ and $|\psi_2\rangle$ are equivalent under local operations when $U|\psi_1\rangle = |\psi_2\rangle$ for a local quantum circuit $U = U_1 \otimes U_2 \otimes \cdots \otimes U_n$ consisting of just single-qubit gates. Show that the following two graph states are equivalent under local operations.



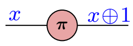
Hint: Use the fact that a local complementation can be done using just local unitaries.

Exercise 2: A graph state has no internal spiders, but a general graph-like diagram does. Show that any graph-like diagram with no inputs (i.e. a state) can be written as a graph-state where each internal spider becomes a post-selection by adapting the arguments from Section 3.4.1 of Picturing Quantum Software.

Exercise 3: Simplify the following circuit to a diagram that has no internal spiders with a $\pm\frac{\pi}{2}$ phase or pairs of internal spiders with a 0 or π phase.



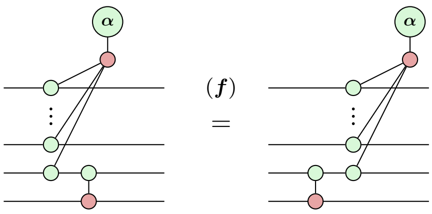
Exercise 4: Show that the phase polynomial representation also works for circuits made of CNOT, Z-phase, and X gates, using the fact that an X gate updates wire labels as follows:



More specifically, give an example circuit, compute its parity map and phase polynomial by wire-labelling, and re-synthesise a smaller circuit.

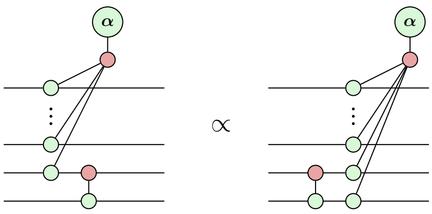
Finally, show that further simplifications are possible: namely expressions of the form $\alpha \cdot y + \beta \cdot (y \oplus 1)$, where y is some XOR of input variables, can be simplified in the phase polynomial, up to global phases.

Exercise 5: We computed the phase gadget form by appealing to the simplification strategy for CNOT circuits from Chapter 3. However, there is a more direct way we can translate CNOT+phase circuits into circuits of phase gadgets, by studying the way that phase gadgets commute with CNOT gates. There are a few cases to think about here. First, is the trivial case where a phase gadget and CNOT gate share no qubits. Obviously these commute. Only slightly harder to see is if a phase gadget appears on the control qubit (i.e. the Z spider) of a CNOT gate. Then, commutation follows from spider fusion:



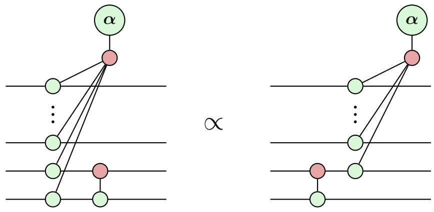
(1)

If the phase gadget overlaps with the target qubit (i.e. the X spider) of a CNOT gate, we can still push a phase gadget through, but it picks up an extra leg:



(2)

Just by reading the equation above in reverse, we can also see what happens when a phase gadget overlaps with both qubits of the CNOT gate. It loses a leg:



(3)

Prove equations (2) and (3). Use the three "phase-gadget walking" equations (1), (2), and (3), as well as the fact that Z-phase gates are equivalent to 1-legged phase gadgets, to show that an n-legged phase gadget has the following decomposition: