Quantum Processes & Computation

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Chapter 1: Introduction

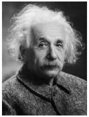
Karma police, arrest this man. He talks in maths.

— *Radiohead, "Karma Police", Oxford, 1997.*

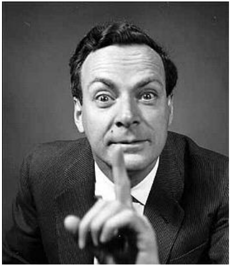
Quantum theory: the standard line

- Quantum theory governs the behaviour of the microscopic world
- You've probably heard from credible sources[1] that it is **weird**, **spooky**, and defies our **natural, classical intuitions**.
- True, it has some 'bugs' from the p.o.v. of classical physics:
  - irreducible non-determinism
  - non-locality
  - incompatible observations
  - ...
- A century of effort went to answering:

Why *is quantum theory so weird, and can we* **fix** *its bugs?*

[1] e.g. 

This produced (basically) two answers



Make even weirder ontology

'Shut up and calculate!'

(e.g. Bohmian mechanics, many worlds, ...)

(Mermin, describing the Copenhagen interpretation)

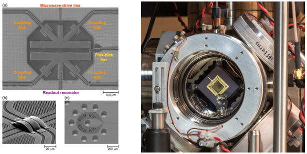
Another, more interesting question

- In the 1980s, a handful of people started to think like software engineers, and ask:

  What if the **bugs** *in quantum theory are actually* **features***?*

- Enter:

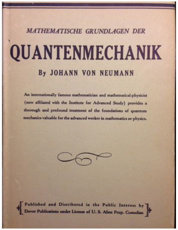
quantum teleportation, communication, cryptography



quantum computation

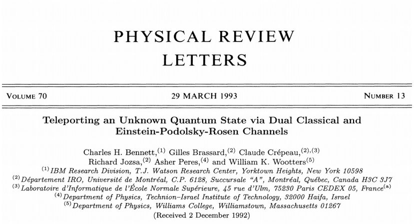
From QT to teleportation

1932 - quantum theory



$\Rightarrow$

1992 - quantum teleportation



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Teleporting an Unknown Quantum State via Dual Classical and Einstein-Podolsky-Rosen Channels

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(Received 2 December 1992)

We'll see that teleportation is **miraculous**...but it's also **totally obvious**.

From QT to teleportation

Q: Why did it take so long?

A: It took 60 years to ask the right question.

Q2: Why is this so hard?

A2: QT needs a better language.

Low-level vs. high-level languages

```
.LC0:
    .string "QUANTUM!"
    .text
    .globl  main
    .type   main, @function
main:
.LFB0:
    .cfi_startproc
    pushq   %rbp
    .cfi_def_cfa_offset 16
    .cfi_offset 6, -16
    movq    %rsp, %rbp
    .cfi_def_cfa_register 6
    subq    $16, %rsp
    movl    $0, -4(%rbp)
    jmp .L2
.L3:
    movl    $.LC0, %edi
    movl    $0, %eax
    call    printf
    addl    $1, -4(%rbp)
.L2:
    cmpl    $4, -4(%rbp)
    jle .L3
    leave
    .cfi_def_cfa 7, 8
    ret
    .cfi_endproc
```

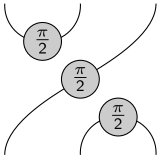
vs.

```
5.times do
  print "QUANTUM!"
end
```

Low-level vs. high-level languages

$$\frac{1}{4}\begin{pmatrix}
-1+i & 1+i & 1+i & -1+i & 1+i & 1-i & 1-i & 1+i \\
1+i & 1-i & 1-i & 1+i & -1+i & 1+i & 1+i & -1+i \\
1+i & 1-i & 1-i & 1+i & 1-i & -1-i & -1-i & 1-i \\
1-i & -1-i & -1-i & 1-i & 1+i & 1-i & 1-i & 1+i \\
1+i & 1-i & 1-i & 1+i & 1-i & -1-i & -1-i & 1-i \\
1-i & -1-i & -1-i & 1-i & 1+i & 1-i & 1-i & 1+i \\
-1+i & 1+i & 1+i & -1+i & 1+i & 1-i & 1-i & 1+i \\
1+i & 1-i & 1-i & 1+i & -1+i & 1+i & 1+i & -1+i
\end{pmatrix}$$

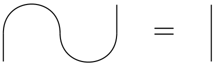
vs.

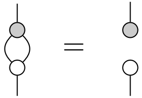


Quantum picturalism

Definition

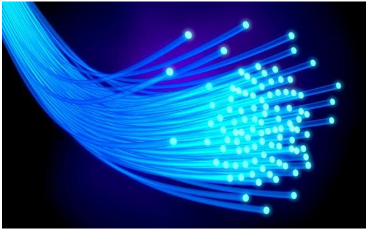
Quantum picturalism refers to the use of diagrams to represent, reason about, and capture essential features and logic of interacting quantum processes.

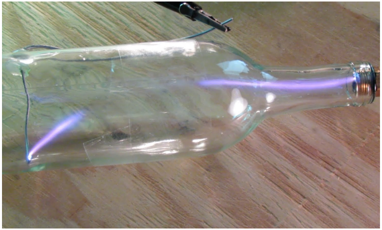




Quantum theory: a warmup

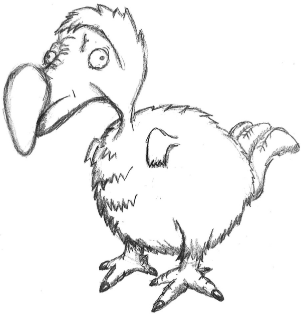
- Typical quantum systems are photons, electrons, etc.





- In this course, we study quantum phenomena that effect all such systems, at an **abstract level**
- So let's focus on a hypothetical, 'alternative' quantum system...

This is Dave.



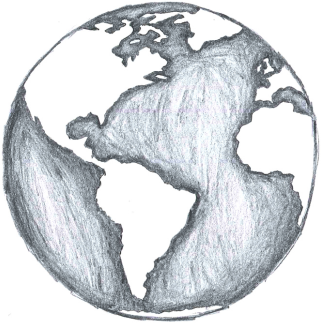
...he's a quantum dodo.

Bits vs. qubits

- Dave's state is given by a *qubit*, the simplest quantum system.

- **Bits:**
  1. admit two states, 0 and 1
  2. can be subjected *any* function
  3. can be read freely, at any time

- **Qubits:**
  1. admit an *entire sphere* of states
  2. can *only be subjected to rotations* of the sphere
  3. can only be accessed by special processes called *quantum measurements*

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Where's Dave?

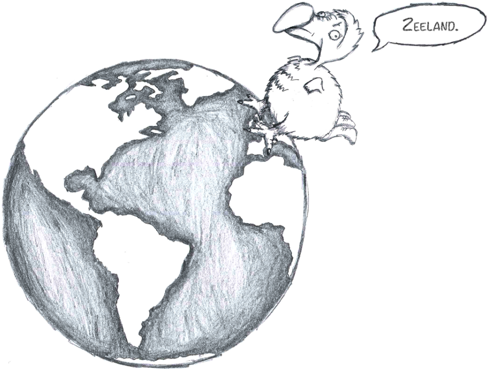
Where's Dave?

The rules:

1. we are only allowed to ask whether a Dave lives at a specific location on Earth or its antipodal location,
2. Dave will always answer 'correctly', i.e. once he gives an answer, that answer becomes correct.

Oxford or New Zealand?

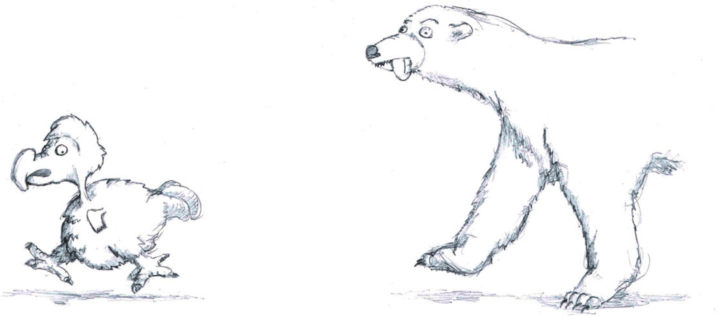
Oxford or New Zealand?



North Pole or South Pole?

North Pole or South Pole?

North Pole or South Pole?



Process theories

- Dave (or rather, a qubit) is just one kind of *system*
- systems undergo processes (e.g. rotations and measurements)
- if we wrap up all the processes which 'fit together' in a theory of physics/logic/computation/etc., we get a **process theory**

The Plan

1. Build the theory of **quantum processes** from scratch,
2. Understand its behaviour using **diagrams**, and
3. Derive some of the most interesting consequences and applications:
   - **quantum communication** (e.g. teleportation and quantum crypto)
   - **quantum computation** (e.g. the factoring algorithm)
   - **quantum foundations** (e.g. quantum non-locality)

Format

- **all material is on the website:**

  www.cs.ox.ac.uk/teaching/courses/2023-2024/quantum

- 24 lectures
- classes in weeks 3, 4, 5, 6, 7, 8
- exam by miniproject (expect a combination of exercise-sheet style and more open-ended problems)