An introduction to *Globular*

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Introduction

Globular is a web-based proof assistant for higher category theory.

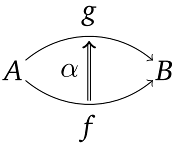
It has many features making it practically useful:

- It's a webpage; nothing to download.
- Graphical point-and-click interface.
- Graphical presentation of morphisms/proofs using *string diagrams*.
- Fully formal; it won't let you make a mistake.
- Download images for inclusion in your paper.
- Link from your paper directly to the formal online proof.
- Share projects privately with collaborators.
- Use existing proofs as lemmas in new proofs.

It's available now at http://globular.science.

Higher categories

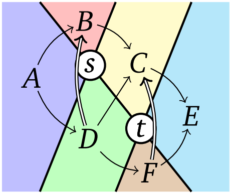
Higher-dimensional categories have *morphisms* between *morphisms*.



Examples: categories, functors, and natural transformations; points, paths, and homotopies; algebraic/coalgebraic theories; freely presented (n-)categories; ...

Graphical notation

Here is a diagram in the 2d graphical notation:



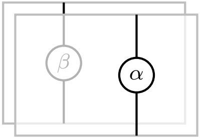
0-morphisms (objects): regions
1-morphisms: wires
2-morphisms: nodes

It is dual to the traditional 'pasting diagram' notation.

Subsumes string diagram notation for monoidal categories (1 object case).

Graphical notation

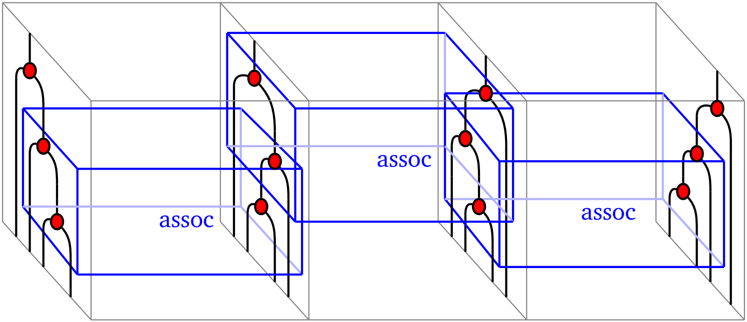
Extends to higher dimensions, e.g. in 3d:



0-morphisms (objects): volumes
1-morphisms: regions
2-morphisms: wires
3-morphisms: nodes

Paradigm: proofs-as-diagrams

Proofs about n-morphisms are diagrams of $n + 1$ morphisms:



Benefit: Proofs can be viewed and transformed (e.g. refactored, simplified) just like any other diagram!

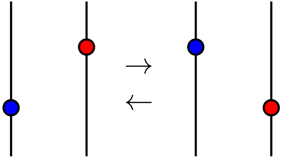
Formalism: semistrict categories

The n-categories we use are *semistrict*. This means:

$$(f \circ g) \circ h = f \circ (g \circ h) \qquad\qquad f \circ 1 = f = 1 \circ f$$

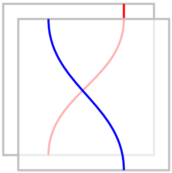
but:

$$(f \circ_1 1_B) \circ_2 (1_{A'} \circ_1 g) \underset{\leftarrow}{\overset{\rightarrow}{}} (1_A \circ_1 g) \circ_2 (f \circ_1 1_{B'})$$

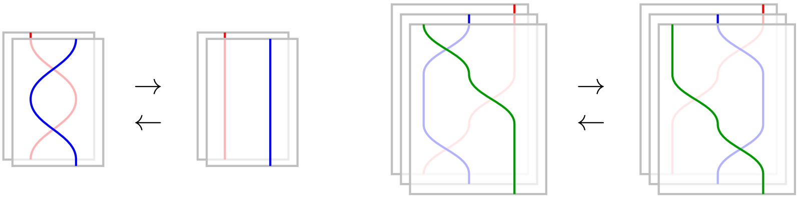


Geometry of interchangers

One dimension higher, interchangers look like crossings:



...and coherence (e.g. invertibility, naturality) makes them *act* like crossings:



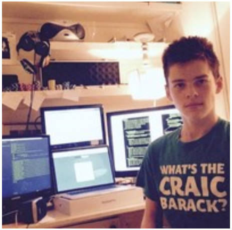
Time to get Globulizing!

Thanks!

These guys did most of the hard stuff... :)

Jamie Vicary

Krzysztof Bar



Caspar Wylie

http://globular.science